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Industry Level and Aggregate Measures of Productivity Growth with Explicit Treatment of Taxes on Products*

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Abstract. We derive industry and aggregate level measures of TFP growth in an open economy as well as the aggregation/decomposition rules from/to the industry level. Net taxes on products in intermediate uses are assumed to be nonzero also at the economy level and different industries are allowed to face different tax rates for the same intermediate inputs. The economy is assumed to be maximizing either the value of deliveries to final demand or value added. In the final demand approach the aggregation equation includes, besides terms representing reallocation of labour and capital, also terms representing reallocation of products in intermediate uses. In the case of Törnqvist indices, if double deflation is used, even reallocation of deliveries final demand between industries contributes to the aggregate TFP growth. For value added there are two alternatives, the approach based on the production possibilities frontier of the industries' value added and the one based on the economy level production function. In the latter case also reallocation of value added between industries contributes to the aggregate TFP growth. When the Divisia and Laspeyres indices are used reallocation terms disappear if tax rates/ prices are identical across industries. In the case of the Törnqvist indices a reallocation term relating to an input only disappears if the growth rate of the input and its value share in the industry's total output are identical across industries. Our results can be generalized to differences in the prices paid by the purchasers caused by any other factor as well. The paper includes an empirical experiment based on the Finnish data.

Key words: Growth accounting, productivity, aggregation
JEL classification: C43, O47

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1. Introduction

In the neoclassical productivity analysis producers are assumed to be maximizing the value of their output in a competitive economy. The necessary conditions of producer equilibrium require, in this framework, the relative prices of inputs and outputs to be identical with the respective marginal rates of transformation. Domar (1961) derived an aggregation rule from the industry level to any aggregate level (the entire economy or a group of industries). The aggregate rate of TFP growth is, in the Domar aggregation, obtained by weighting each industry-level rate of TFP growth "by the ratio of the output of its industry to the value of the final product of the sector." Hulten (1978) proved the Domar aggregation rule in the case of a closed economy, in which prices paid by the users are equal to those received by the producers and all industries pay identical prices for their primary inputs. Jorgenson, Gollop and Fraumeni (1987) in their seminal contribution to productivity measurement developed an aggregation rule in which neither of these assumptions is needed. It is however based on the aggregate production function. Jorgenson (1969) introduced methodology based on aggregate production possibility frontier. This methodology is used by Jorgenson and Stiroh (2000) and Jorgenson (2001).

Taxes on products, as well as respective subsidies, often constitute a wedge between the prices paid by the purchaser, i.e. user, and the one received by the producer. Especially in economies with value added taxes the difference between these two sets prices can be considerable and the prices paid for the same product may differ across industries. This is because all the producers are not liable to charge value added tax on their sales, if any, and therefore cannot deduct the value added taxes paid for their intermediate inputs from the VAT invoiced to their customers. Another reason for the differences in the tax margins across the industries is the fact the products used as intermediate inputs actually consist of baskets of products with possibly different tax rates. Also trade and transport margins, if they are included in the purchasers' prices and not treated as a separate inputs can lead to different purchaser's prices for different users. Here these margins are assumed to be treated as a separate input. The results can however, in principle, be interpreted to cover the trade and transport margins as well.

In productivity analysis it is rather natural to value the output and inputs from the producer's point of view. E.g. JGF (1987) value outputs at basic prices and intermediate inputs at purchaser's prices less trade and transport margins. However, at the economy level the measure of the performance of an economy is often GDP at market prices. This would require the deliveries to final demand to be valued at purchasers' price. Diewert (2005) has suggested the ensuing discrepancy between he GDP at market prices and total value added at basic prices to be rectified by treating product taxes as a separate "industry", with its own value added. But he agrees with Jorgenson and Griliches (1972) that for output basic prices are the prices best suited to productivity accounts.

Gollop (1987) derived the industry and economy level TFP measures as well as the respective aggregation/decomposition rules in an economy maximising the aggregate value of the deliveries to final demand (at basic prices) on the one hand and for an economy maximising the value of the aggregate value added on the other, with the respective production possibilities frontiers as starting points. Unlike Hulten (1978), Gollop (1987) did not assume either closed economy or identical prices received and paid for products used as intermediate inputs. Gollop (1987) did however assume that 1) at the economy level product taxes less subsidies on intermediate inputs cancel out and 2) all the industries pay identical prices for a product used as intermediate input.[1] Also Aulin-Ahmavaara (2003) discussed the need to take into account the product taxes and subsidies on intermediate inputs in productivity measurement based on national accounts.

A natural candidate for the discrete approximation of the continuous time Divisia index is the Törnqvist or translog productivity index and respective quantity indexes for inputs and outputs used by Christensen and Jorgenson (1970) as well as by JGF (1987). Diewert (1976) has shown Törnqvist index to be exact for translog aggregator function and Caves et al (1982) have presented strong economic support for the use Törnqvist productivity index. On the other hand Hill (2001) argues that the choice between different superlative indices cannot be made solely on the basis of the economic approach. Milana (2005) concludes that the index numbers normally considered as superlative in fact are hybrids and recommends constructing a range of alternative index numbers (including those that are not superlative). The nonsuperlative Laspeyres indexes were used by Jorgenson and Griliches (1967) and later also e.g. by Stenbæk and Sørensen (2004) in their study on the productivity development in Denmark.

In the present paper different approaches to the measurement and aggregation of TFP growth are studied, with an emphasis on the implications of the existence of taxes and subsidies on products. Unlike Gollop (1987) we do not assume taxes and subsidies on products in intermediate uses to cancel out at the economy level. Neither do we assume identical tax rates in all the industries for a product used as intermediate input. Following Jorgenson and Griliches (1967) we start with the accounting identities and derive, in sections 2-4, the industry and economy level TFP-measures based on different definitions of output as well as the respective aggregation rules. We outline, in line with Gollop (1987), with appropriate modifications, the interpretation of our results in terms of the neoclassical theory of production and producer behaviour. In section 5 we derive the aggregation rules from the industry level to an intermediate level in order to clarify the significance of the

[1] This is obvious for instance from his equation (19).

level aggregation in productivity measurement. Our theoretical system is based on the continuous time Divisia indices. In sections 6 and 7 we derive respective systems based on the Laspeyres indices on the one hand and on the Törnqvist indices on the other. In section 8 we report the preliminary results from the calculations based on the Finnish input output tables from years 1999 and 2000 and discuss the choices to be made in the empirical calculations.

2. Industry level measures of TFP growth

We start the derivation of TFP measures, in line with Jorgenson and Griliches (1967), from the following accounting identity:

(1) $$\mathbf{q}'\mathbf{z} = \mathbf{p}'\mathbf{v}$$

where $\mathbf{q}$ is the price vector of outputs

$\mathbf{z}$ is the vector of output quantities

$\mathbf{p}$ is the price vector of inputs and

$\mathbf{v}$ is the vector of input quantities.

The rate of total factor productivity growth is defined as the difference between the growth rates of outputs and inputs:

(2) $$d\log t = \sum_i \alpha_i d\log z_i - \sum_j \beta_j d\log v_j = \sum_j \beta_j d\log q_i - \sum_i \alpha_i d\log p_i,$$

where α_i is the share of the i th output in total revenue and β_j the share of the j th input in total cost and $d\log y$ is the logarithmic time derivative of the variable y. This measure can be given an economic interpretation as the shift of the production function when a production function with constant returns to scale is assumed and all the relevant assumptions concerning markets and producer behaviour are made.

Following the SNA93 (ISWGNA, 1993), and assuming that capital and labour compensation together cover the entire value added, the accounting identity for an industry with only one type of output is defined as follows:

(3) $$q_j Q_j = \sum_i p_{ij} M_{ij} + \sum_i p_{ij}^M M_{ij}^M + \sum_k p_k^K K_{kj} + \sum_l p_l^L L_{lj}\ .$$

The following notation is used

Q_j quantity of the output of the j th industry

q_j basic price of the output of the j th industry

M_{ij} quantity of the output of the i th industry used as intermediate input by the j th industry

p_{ij} purchaser's price (without trade and transport margins) paid by the j th industry for a unit of the output of the i th industry it uses as intermediate input

d_{ij} net[2] taxes on products[3] per unit of output of industry i used as input in industry j

M_{ij}^M quantity of the i th imported product used as intermediate input by the j th industry

q_i^M c.i.f. price of i th imported product

p_{ij}^M purchaser's price (without trade and transport margins) paid by the j th industry for a unit of the i th imported product it uses as intermediate input

d_{ij}^M net taxes on products per unit of imported product i used as input in industry j.

M_{ij}^M quantity of the i th imported product used by the j th industry as intermediate input

p_{kj}^K price paid by the j th industry for the capital input of category k

K_{kj} quantity of the capital input of category k used by the j th industry

p_{lj}^L price paid by the j th industry for the labour input of category l

L_{lj} quantity of the labour input of category l used by the j th industry.

When trade and transport margins are treated as separate inputs then the only difference between basic prices and purchasers' prices are taxes and subsidies on products. Thus we can also write

(4) $$q_j Q_j = \sum_i (1+d_{ij}) q_i M_{ij} + \sum_i (1+d_{ij}^M) q_i^M M_{ij}^M + \sum_k p_k^K K_{kj} + \sum_l p_l^L L_{lj} .$$

The accounting identities above are based on gross output. The two other possible concepts of output are the so called sectoral output and value added. For sectoral output the accounting equation should be written as follows:

[2] Taxes on products are net of similar subsidies. It is of course also possible to interpret this term to represent a price differential caused by some other factors.

$$
(5)\quad \begin{aligned} q_j\overline{Q}_j &= q_jQ_j - q_jM_{jj} \\ &= d_{jj}M_{jj} + \sum_{i\neq j}(1+d_{ij})q_iM_{ij} + \sum_i (1+d_{ij}^M)q_i^M M_{ij}^M + \sum_k p_k^K K_{kj} + \sum_l p_l^L L_{lj} \end{aligned}
$$

with sectoral output denoted by $\overline{Q}_j$. And in for value added as follows

$$
(6)\quad \begin{aligned} v_jV_j &= q_jQ_j - \sum_i q_iM_{ij} - \sum_i d_{ij}q_iM_{ij} - \sum_i q_i^M M_{ij}^M - \sum_i d_{ij}^M q_i^M M_{ij}^M \\ &= \sum_k p_k^K K_{kj} + \sum_l p_l^L L_{lj} \end{aligned}.
$$

Applying the formula in equation (2) to the accounting identity in equation (4) gives the rate of industry level TFP change for gross output

$$
(7)\quad \begin{aligned} d\log t_j &= (q_jQ_j)^{-1}[q_jQ_j d\log Q_j - \sum_i (1+d_{ij})q_iM_{ij}d\log M_{ij} \\ &- \sum_i (1+d_{ij}^M)q_i^M M_{ij}^M d\log M_{ij}^M - \sum_k p_{kj}^K K_{kj} d\log K_{kj} - \sum_l p_{lj}^L L_{lj} d\log L_{lj})] \end{aligned}.
$$

The same formula is obtained, following e.g. Gollop (1987) by taking the total logarithmic derivative of the industry level production function

$$
(8)\quad Q_j = h^j(L_j, K_j, M_{1j}, M_{2j}, \ldots M_{jj}, \ldots M_{nj}, M_{1j}^M, M_{2j}^M, \ldots M_{uj}^M, t)
$$

and substituting the conditions of producer equilibrium into the result.

Respectively the rate of TFP growth and the production function in the case of “sectoral output” are the following:

$$
(9)\quad \begin{aligned} d\log\bar{t}_j &= (q_j\overline{Q}_j)^{-1}[q_j\overline{Q}_j d\log\overline{Q}_j - d_{jj}q_jM_{jj}d\log M_{jj} - \sum_{i\neq j}(1+d_{ij})q_iM_{ij}d\log M_{ij} \\ &- \sum_i (1+d_{ij}^M)q_i^M M_{ij}^M d\log M_{ij}^M - \sum_k p_{kj}^K K_{kj} d\log K_{kj} - \sum_l p_{lj}^L L_{lj} d\log L_{lj})] \end{aligned}
$$

and

---

[3] “Taxes on products” is the term used in the SNA93. It is used also here. The corresponding term in the SNA68 was “commodity taxes”.

(10) $\overline{Q}_j = \overline{h}^j(L_j, K_j, M_{1j}, M_{2j}, \ldots M_{jj}, \ldots M_{nj}, M_{1j}^M, M_{2j}^M, \ldots M_{uj}^M, t)$.

Although the production function in equation (10) has the same arguments as the one in equation (8), it is not the same function. The marginal revenue product of the intermediate input from the industry itself has now, in the producer equilibrium, to be equal to the net rate of product taxes paid on it.

And finally for value added the rate of industry level TFP growth is

(11) $d\log t_j^v = (v_j V_j)^{-1}[v_j V_j d\log V_j - \sum_k p_{kj}^K K_{kj} d\log K_{kj} - \sum_l p_{lj}^L L_{lj} d\log L_{lj}]$

The industry is now assumed to be maximizing its value added with the following value added function

(12) $V_j = V^j(L_j, K_j, t)$.

The existence of this kind of industry level value added function that links technological change exclusively to real value added and primary inputs, implies that the industry level production function is value-added separable:[4]

(13) $Q_j = g^j[V^j(L_j, K_j, t), M_{1j}, M_{2j}, \ldots M_{nj}, M_{1j}^M, M_{2j}^M, \ldots M_{uj}^M]$.

The relationship between the measure based on the sectoral output and the one based on gross output is obtained by taking logarithmic derivative of both sides of the first expression of equation (5)

(14) $q_j \overline{Q}_j d\log \overline{Q}_j = q_j Q_j d\log Q_j - q_j M_{jj} d\log M_{jj}$.

Substituting the result in equation (9) gives, by equation (7)

(15) $d\log \overline{t}_j = (q_j \overline{Q}_j)^{-1}(q_j Q_j) d\log t_j$.

---

[4] In this case the volume index of value added and the measure of TFP growth are “path independent”. The, possibly path dependent, volume and price indices of value added, however do exist even if the production function is not separable. For a discussion of this see OECD (2001) Productivity Manual and the sources mentioned in it.

Taking the logarithmic derivative of the first expression in equation (6) produces:

(16)
$$v_j V_j d\log V_j = q_j Q_j d\log Q_j - \sum_i (1+d_{ij}) q_i M_{ij} d\log M_{ij} - \sum_i (1+d_{ij}^M) q_i^M M_{ij}^M d\log M_{ij}^M \cdot$$

Substituting this in equation (11) gives in view of equation (7) the following relationship between the industry level measures based on the value added on the hand and on the gross output on the other:

(17)
$$d\log t_j^v = (v_j V_j)^{-1}(q_j Q_j) d\log t_j \, .$$

Remark 1. If the industry-level measure of output is the "sectoral output" the input from industry itself still has to be included in the production function as well as in the calculation of the rate of TFP change. Its weight is the share of net product taxes on this input. (equation 9).

3. Economy level measures of TFP growth

At the economy level the entire economy is treated as a single unit of production. The difference to the industry level, in our case, is that the output of an economy consists of several products. Since the economy is treated as a single unit of production we obviously have to assume that all the industries face identical prices for their inputs. The quantity Z and the price p^Z of an input at the economy level can in this case, in line with JGF (1987), be defined, on the basis of the industry level quantities and prices, as follows:

(18)
$$\sum_j p_j^Z Z_j = p^Z \sum_j Z_j = p^Z Z \, .$$

The economy level accounting identity based on the gross output now is

(19)
$$\sum_j q_j Q_j = \sum_i (q_i + d_i) M_i + \sum_i (q_i^M + d_i^M) M_i^M + \sum_k p_k^K K_k + \sum_l p_l^L L$$

At the economy level the deliveries to final demand of domestic output at basic prices constitute the counterpart of sectoral output at industry level

$$\begin{aligned}\sum_j q_j Y_j &= \sum_j q_j Q_j - \sum_{j,i} q_j M_{ji} \\ &= \sum_i d_i q_i M_i + \sum_i (1+d_i^M) q_i^M M^M{}_i + \sum_k p_k^K K_k + \sum_l p_l^L L\end{aligned} \quad (20)$$

Our third measure of economy level output is the sum of values of industries' value added:

$$\begin{aligned}\sum_j v_j V_j &= \sum_j q_j Q_j - \sum_j \sum_i (1+d_{ij}) q_i M_{ij} - \sum_j \sum_i (1+d_{ij}^M) q_i^M M^M_{ij} \\ &= \sum_k p_k^K K_k + \sum_l p_l^L L \ .\end{aligned} \quad (21)$$

The aggregate production function is not assumed to exist, hence the different prices of the value added as well as for the intermediate inputs in different industries. In this all the industries are no assumed to be paying identical prices for their intermediate inputs, only for their capital and labour inputs.

From the equations (21) and 20 we obtain:

$$\sum_j q_j Y_j = \sum_j v_j V_j + \sum_j \sum_i d_{ij} q_i M_{ij} + \sum_j \sum_i (1+d_{ij}^M) q_i^M M^M_{ij} \ . \quad (22)$$ [5]

Remark 2. The sum of values of the industries value added and the sum of the values of the deliveries to final demand (at basic prices) are equal if and only if there are no imported intermediate inputs and the aggregate value of taxes or subsidies on products in intermediate uses equals zero (equation 22).

Applying the formula in equation (2) to the accounting equation (19) gives the rate of TFP growth for the gross output of the economy

$$\begin{aligned}d\log T = {}& (\sum_j q_j Q_j)^{-1} [\sum_j q_j Q_j d\log Q_j - \sum_i (1+d_i) q_i M_i d\log M_i \\ & - \sum_i (1+d_i^M) q_i^M M^M{}_i d\log M_i - \sum_k p_k^K K_k d\log K_k - \sum_l p_l^L L_l d\log L_l].\end{aligned} \quad (23)$$

[5] Another option would be to use purchasers' prices, or rather purchasers' prices minus trade and transport margins. This would mean valuation from the users' point of view rather than from the producer's point of view. In case the valuation were based on the purchaser's prices, then equation (22) should be written as follows $\sum_j p_j Y_j = \sum_j v_j V_j + \sum_j \sum_i q_i^M M^M_{ij} + \sum_j \sum_i d_{ij} M_{ij} + \sum_j \sum_i d_{ij}^M M^M_{ij} + \sum_j d_j Y_j$. The three last terms of this expression together include all the net taxes on products except those paid for imported products in final uses.

The maximum value of gross output (γ) can be expressed as a function of industries' gross output (Q_j), primary inputs (K, L), domestically produced intermediate inputs (M_i), imported intermediate inputs (M_i^M), and time (t):[6]

(24) $$\gamma = F(Q_1, Q_2, \ldots Q_n, L, K, M_1, M_2, \ldots M_n, M_1^M, M_2^M, \ldots M_u^M, t).$$

The value of gross output is maximized subject to fixed supplies of domestic capital and labour inputs, market equilibrium and linearly homogeneous industry level production functions in equation (8). The function F is homogenous of degree minus one in industries' gross output and of degree one in the rest of the variables (the input variables), except time, t. The rate of aggregate TFP growth is obtained by setting $\gamma = 1$, taking the total logarithmic derivative of F with respect to time and substituting the producer equilibrium conditions into the result.

Applying the formula in equation (2) to the accounting equation (20) gives the rate of TFP growth based on the deliveries to final demand of domestic output

(25) $$\begin{aligned} d\log\overline{T} = (\sum_j q_j Y_j)^{-1}[\sum_j q_j Y_j d\log Y_j - \sum_i d_i q_i M_i d\log M_i - \sum_i (1+d_i^M) q_i^M M^M{}_i d\log M_i \\ - \sum_k p_k^K K_k d\log K_k - \sum_l p_l^L L_l d\log L_l]. \end{aligned}$$

In this case the economy is maximising the value of deliveries to final demand μ

(26) $$\mu = H(Y_1, Y_2, \ldots Y_n, L, K, M_1, M_2, \ldots, M_n, M_1^M, M_2^M, \ldots, M_u^M, t).$$

subject to fixed supplies of domestic capital and labour inputs, market equilibrium and linearly homogeneous industry level production functions in equation (8). The domestic intermediate inputs appear in equation (26) but their role is different from the one in (24). The second term in the square brackets in equation (25) disappears regardless of the rates of growth of individual intermediate inputs if there are no taxes or subsidies on products in intermediate uses. On the other hand if $d_i \neq 0$ for some domestic intermediate inputs the value of the term depends on the rates of growth of individual intermediate inputs.

[6] The derivation of TFP-measures starting from the production frontiers and production functions outlined in this paper is mainly based on our interpretation of Gollop (1987). It is also discussed in Aulin-Ahmavaara (2004).

Remark 3. When the economy level production possibilities frontier is based on the deliveries to final demand of domestic products then variables representing domestic intermediate inputs have to be included in it. Accordingly, also terms representing the rates of change of domestic intermediate inputs weighted by the shares of net product taxes paid on them have to be included in the formula for the rate of economy level TFP growth (equation 25).

Applying the formula in equation (2) to the latter expression in the accounting equation (21) gives the rate gives the rate of TFP growth based on the industries' value added

(27) $$d\log T^{v} = (\sum_{j} v_j V_j)^{-1}[(\sum_{j} v_j V_j d\log V_j - \sum_{k} p_k^K K_k d\log K_k - \sum_{l} p_l^L L_l d\log L_l)].$$

In this case the maximum value of the aggregate value added (λ) is a function of all quantities of industries' value added (V_j), aggregate labour (L) and capital (K) inputs, and time (t):

(28) $$\lambda = G(V_1, V_2, \ldots V_n, L, K, t).$$

The economy is maximising λ subject to linearly homogeneous value added functions in equation (12) as well as market equilibrium conditions, and aggregate supplies of capital and labour.

The relationships between the different economy level measures can be easily established by following the familiar procedure used at the industry level. From the first expression in equation (20) we get

(29) $$\sum_{j} q_j Y_j d\log Y_j = \sum_{j} q_j Q_j d\log q_j Q_j - \sum_{j,i} q_j M_{ji} d\log M_{ji}.$$

Substituting this into equation (25) gives, by equation (23), the following relationship between the economy level measures based on the gross output on the one hand and on the deliveries to final demand on the other:

(30) $$d\log \overline{T} = (\sum_{j} q_j Y_j)^{-1}(\sum_{j} q_j Q_j) d\log T.$$

The relationship between the value added based measure and the measure based on the deliveries to the final demand of domestic output is somewhat less simple. From equation (21) we get

$$(31)\quad \begin{aligned}&\sum_j q_j Y_j d\log Y_j = \sum_j v_j V_j d\log V_j \\ &+\sum_j \sum_i d_{ij} q_i M_{ij} d\log M_{ij} + \sum_j \sum_i q_i^M M^M{}_i d\log M_i^M + \sum_j \sum_i d_{ij}^M q_i^M M_{ij}^M d\log M_{ij}^M\end{aligned} .$$

Substituting this into equation (25) leads by equation (27) to the following relationship

$$(32)\quad \begin{aligned}&d\log \overline{T} = (\sum_j q_j Y_j)^{-1}[(\sum_j v_j V_j) d\log T^v \\ &\quad + (\sum_j \sum_i d_{ij} q_i M_{ij} d\log M_{ij} - \sum_i d_i q_i M_i d\log M_i) \\ &\quad + (\sum_j \sum_i d_{ij}^M q_i^M M_{ij}^M d\log M_{ij}^M - \sum_i d^M{}_i q_i^M M_i^M d\log M_i^M)]\end{aligned} .$$

If the price of an input Z is identical for all the industries, i.e. if $p_j^Z = p^Z$ for all values of j, then it follows directly from the definition in equation (18) that

$$(33)\quad \sum_j p_j^Z Z_j d\log Z_j = \sum_j p^Z dZ_j = p^Z dZ = p^Z Z \; d\log Z \; .$$

Therefore the second line and third line in equation (32) disappear if the tax rates of intermediate inputs are identical in all industries.

Remark 4. If all the industries pay identical prices for their intermediate inputs, both domestic and imported, the difference between the two economy level rates of growth depends only on the ratio of the value of the deliveries to final demand to sum of the industries' value added (equation 32). If the prices paid by different industries for intermediate inputs are not identical, then the difference between the two economy level rates of TFP growth depends also on the reallocation of these inputs across industries (equation 32).

4. Aggregation from the industry level to the economy level

To establish the relationship between the industry level measure based on gross output, equation (7) and the economy level measure based on the deliveries to final demand, equation (25), we multiply both sides of the equation (7) for each industry by the value of the industry's output. Summing up across industries produces:

$$
(34)\quad \begin{aligned}\sum_j q_j Q_j d\log t_j &= \sum_j q_j Q_j d\log Q_j - \sum_j\sum_i (1+d_{ij}) q_i M_{ij} d\log M_{ij} \\ &-\sum_j\sum_i (1+d^M_{ij}) q^M_i M^M_{ij} d\log M^M_{ij} - \sum_j\sum_k p^K_{kj} K_{kj} d\log K_{kj} - \sum_j\sum_l p^L_{lj} L_{lj} d\log L_{lj}.\end{aligned}
$$

Dividing both sides of (34) by the aggregate value of the deliveries to final demand $\sum q_j Y_j$ and subtracting the result from both sides of equation (25) yields

$$
(35)\quad \begin{aligned} d\log\overline{T} &= (\sum_j q_j Y_j)^{-1}[\sum_j q_j Q_j d\log t_j \\ &+\sum_j\sum_i d_{ij} q_i M_{ij} d\log M_{ij} - \sum_i d_i q_i M_i d\log M_i \\ &+\sum_j\sum_i d^M_{ij} q^M_i M^M_{ij} d\log M^M_{ij} - \sum_i d^M_i q^M_i M^M_i d\log M^M_i \\ &+\sum_j\sum_k p^K_{kj} K_{kj} d\log K_{kj} - \sum_k p^K_k K_k d\log K_k \\ &+\sum_j\sum_l p^L_{lj} L_{lj} d\log L_{ij} - \sum_l p^L_l L_l d\log L_l]\end{aligned}
$$

The contribution of the industry level rates of TFP growth is represented by the first term in square brackets. The rest of the terms represent the contribution of the reallocation of the inputs by industry.

Remark 5. The rate of economy level TFP growth based on the deliveries of domestic output to final demand consists of (equation 35) 1) the weighted sum of the industry-level rates of TFP-growth based on the industries' gross outputs with the ratios of the industries' outputs to the total value of deliveries to final demand as weights and 2) terms that reflect reallocation of capital, labour and intermediate inputs, both domestic and imported, across industries. However, if all the industries pay identical price for an individual input, the rate of the economy level TFP-growth does not depend on the reallocation of that input across industries (equation 35 and 33).

To establish the relationship between the industry level measures based on the industries' value added, in equation (11) and the economy level measure based on the industries' value added, equation (27) we multiply both sides of equation (11) by the ratio $(v_j V_j)(\sum_j v_j V_j)^{-1}$. Summing up across industries and subtracting the result from both sides of equation (27) produces

$$
\begin{aligned}
(36)\qquad d\log T^{v} = {} & (\sum_j v_j V_j)^{-1}[\sum_j v_j V_j d\log t_j^{v} \\
& + \sum_j \sum_k p_{kj}^{K} K_{kj} d\log K_{kj} - \sum_k p_k^{K} K_k d\log K_k \,. \\
& + \sum_j \sum_l p_{lj}^{L} L_{lj} d\log L_{ij} - \sum_l p_l^{L} L_l d\log L_l \,]
\end{aligned}
$$

This equation gives the economy level measure based on the value added approach in terms of the industry level measures based on the value added.

Substituting the relationship between the industry level rate of TFP growth based on the gross output given in equation (15) into equation (36) produces an expression for the relationship between the aggregate value-added based rate of TFP-growth and the respective industry level rates expressed in terms of total output:

$$
\begin{aligned}
(37)\qquad d\log T^{v} = {} & (\sum_j v_j V_j)^{-1}[\sum_j q_j Q_j d\log t_j \\
& + (\sum_j \sum_k p_{kj}^{K} K_{kj} d\log K_{kj} - \sum_k p_k^{K} K_k d\log K_k) \,. \\
& + (\sum_j \sum_l p_{lj}^{L} L_{lj} d\log L_{ij} - \sum_l p_l^{L} L_l d\log L_l)]
\end{aligned}
$$

Remark 6. The rate of economy level TFP growth based on the industries' value added consists of (equation 37) 1) the weighted sum of the industry-level rates of TFP-growth based on the industries' gross outputs with the ratios of the industries' output to the sum of the industries' value added as weights and 2) terms that reflect reallocation of capital and labour across industries. However the economy level measure based on the production possibility frontier does not depend on the reallocation of intermediate input with different tax rates across industries.

5. Intermediate level measures of TFP growth and aggregation to the intermediate level

Next we shall consider the case in which a group of industries, which of course can cover the entire economy, is treated as a single unit of production that produces only one type of output. The accounting identity based on gross output is now written as follows

$$
(38)\qquad q_S Q_S = (1+d_{SS}) q_S M_{SS} + \sum_{i\notin S} (1+d_{iS}) q_i M_{iS} + \sum_i (1+d_{iS}^{M}) q_i^{M} M_{iS}^{M} + \sum_k p_{kS}^{K} K_{kS} + \sum_l p_{lS}^{L} L_{lS} \,.
$$

And the one for "sectoral output" i.e. on the output delivered outside the group of industries as follows

$$
(39) \qquad \begin{aligned} q_S\overline{Q}_S &= q_S Q_S - q_S M_{SS} \\ &= d_{SS} q_S M_{SS} + \sum_{i \notin S} (1 + d_{iS}) q_i M_{iS} + \sum_i (1 + d_{iS}^M) q_i^M M_{iS}^M + \sum_k p_{kS}^K K_{kS} + \sum_l p_{lS}^L L_{lS} \end{aligned}
$$

And finally for value added

$$
(40) \qquad \begin{aligned} v_S V_S &= q_S Q_S - (1 + d_{SS}) q_S M_{SS} - \sum_{i \notin S} (1 + d_{iS}) q_i M_{iS} - \sum_i (1 + d_{iS}^M) q_i^M M_{iS}^M \\ &= \sum_k p_{kS}^K K_{kS} + \sum_l p_{lS}^L L_{lS} \end{aligned} .
$$

The derivation of different rates of aggregate TFP-growth and their mutual relationships as well as the economic interpretation of the measures is analogous to the one at the industry and economy levels. The formula for the rate based on the gross output accordingly is

$$
(41) \qquad \begin{aligned} d\log T_S = (q_S Q_S)^{-1} [q_S Q_S d\log Q_S - (1 + d_{SS}) q_S M_{SS} d\log M_{SS} - \sum_{i \notin S} (1 + d_{iS}) q_i M_{iS} d\log M_{iS} \\ - \sum_i (1 + d_{iS}^M) q_i^M M_{iS}^M d\log M_{iS} - \sum_k p_{kS}^K K_{kS} d\log K_{kS} - \sum_l p_{lS}^L L_{lS} d\log L_{lS}]. \end{aligned}
$$

And the one based on the value added analogously to the industry level is

$$
(42) \qquad d\log T_S^v = (v_S V_S)^{-1} [v_S V_S d\log V_S - \sum_k p_{kS}^K K_{kS} d\log K_{kS} - \sum_l p_{lS}^L L_{lS} d\log L_{lS}] .
$$

This rate of TFP growth based on the aggregate value added V_S of the group of industries obviously implies that the value added function does exist and accordingly that the value added functions of all the industries in the group are identical up to a scalar multiple.[7]

The relationship between the rate of TFP growth based on the gross output and the one based on value added is also analogous to the industry level:

$$
(43) \qquad d\log T_S^v = (v_S V_S)^{-1} (q_S Q_S) d\log T_S .
$$

[7] For more on this see JGF(1987) and the sources given in it.

To establish the relationship between the industry level measure based on gross output, equation (7) and the aggregate level measure based on the gross output, equation (41), we first multiply both sides of equation (7) by the value of the industry's output and sum up over industries in the group S to obtain:

$$
\begin{aligned}
\sum_{j\in S} q_j Q_j d\log t_j &= \sum_{j\in S} q_j Q_j d\log Q_j \\
&- \sum_{j\in S}\sum_{i\in S}(1+d_{ij})q_i M_{ij} d\log M_{ij} - \sum_{j\in S}\sum_{i\notin S}(1+d_{ij})q_i M_{ij} d\log M_{ij} \\
&- \sum_{j\in S}\sum (1+d_{ij}^M)q_i^M M_{ij}^M d\log M_{ij}^M \\
&- \sum_{j\in S}\sum_k p_{kj}^K K_{kj} d\log K_{kj} - \sum_{j\in S}\sum_l p_{lj}^L L_{lj} d\log L_{lj}.
\end{aligned} \tag{44}
$$

The relationship between the aggregate measure at the intermediate level and the industry level measures is obtained by dividing both sides of equation (44) by the total value of the output of the industries in the group S , $q_S Q_S$, and subtracting the result from both sides of equation (41):

$$
\begin{aligned}
d\log T_s &= (q_S Q_S)^{-1}[\sum_{j\in S} q_j Q_j d\log t_j - (\sum_{j\in S} q_j Q_j d\log Q_j - q_S Q_S d\log Q_S) \\
&+ \sum_{j\in S}\sum_{i\in S}(1+d_{ij})q_i M_{ij} d\log M_{ij} - (1+d_{SS})q_S M_{SS} d\log M_{SS} \\
&+ \sum_{j\in S}\sum_{i\notin S} d_{ij} q_i M_{ij} d\log M_{ij} - \sum_{i\notin S} d_{iS} q_i M_{iS} d\log M_{iS} \\
&+ \sum_{j\in S}\sum_i d_{ij}^M q_i^M M_{ij}^M d\log M_{ij}^M - \sum_i d_{iS}^M q_i^M M_{iS}^M d\log M_{iS} \\
&+ \sum_{j\in S}\sum_k p_{kj}^K K_{kj} d\log K_{kj} - \sum_k p_{kS}^K K_{kS} d\log K_{kS} \\
&+ \sum_{j\in S}\sum_l p_{lj}^L L_{lj} d\log L_{lj} - \sum_l p_{lS}^L L_{lS} d\log L_{lS}].
\end{aligned} \tag{45}
$$

Remark 7. The aggregate rate of TFP change for a group of industries (or an economy as a whole) depends besides the rates of TFP growth of the individual industries also on the reallocation of output and inputs across the industries. (Equation 45 and 33).

To establish the relationship between the industry level measures based on industries' value added, in equation (11) and the respective aggregate measure at the intermediate level (or at the level of the entire economy), equation (42), we multiply both sides of (11) by the ratio

$(v_j V_j)(v_S V_S)^{-1}$. Summing up across industries and subtracting the result from both sides of equation (42) produces

$$
\begin{aligned}
(46)\qquad & d\log T_S^v = (v_S V_S)^{-1}[\sum_{j\in S} v_j V_j d\log t_j \\
& + (v_S V_S d\log V_S - \sum_{j\in S} v_j V_j d\log V_j) \\
& + (\sum_{j\in S}\sum_k p_{kj}^K K_{kj} d\log K_{kj} - \sum_k p_{kS}^K K_{kS} d\log K_{kS})) \\
& + (\sum_{j\in S}\sum_l p_{lj}^L L_{lj} d\log L_{ij} - \sum_l p_{lS}^L L_{lS} d\log L_{iS})].
\end{aligned}
$$

The second term on the RHS can be further developed by taking the logarithmic derivatives from the first expressions in equation (6) for the industry level value added and equation (40) for the intermediate level value added. Summing up the former across industries and subtracting the result from the latter produces

$$
\begin{aligned}
(47)\qquad & v_S V_S d\log v_S V_S - \sum_{j\in S} v_j V_j d\log V_j = -(\sum_{j\in S} q_j Q_j d\log Q_j - q_S Q_S d\log Q_S) \\
& + (\sum_{j\in S, i\in S} (1+d_{ij}) q_i M_{ij} d\log M_{ij} - (1+d_{SS}) q_S M_{SS} d\log M_{SS}) \\
& + (\sum_{j\in S, i\notin S} d_{ij} q_i M_{ij} d\log M_{ij} - \sum_{i\notin S} d_{iS} q_i M_{iS} d\log M_{iS}) \\
& + (\sum_{j\in S, i} d_{ij}^M q_i^M M_{ij}^M d\log M_{ij}^M - \sum_i d_{iS}^M q_i^M M_{iS}^M d\log M_{iS}^M)
\end{aligned}
$$

Substituting the RHS of equation (47) in equation (46) shows, in view of equation (45), that both intermediate level measures of rate of TFP growth actually include the same reallocation terms.

Remark 8. The aggregate rate of TFP change for a group of industries (which of course can also cover the entire economy) based on the value added of the group depends besides the rates of TFP growth of the individual industries also on the reallocation of value added and capital and labour inputs across industries within the group (equation 46). The term representing the reallocation of value added depends on the reallocation of outputs and intermediate inputs across industries exactly in the same way as the aggregate rate of TFP growth based on the gross output (equations 45 and 47).

6. Application Based on Laspeyres indexes

The Laspeyres quantity indices of industry level outputs and inputs are defined as follows:

Output $$Q_j = \frac{q_j^0 Q_j^1}{q_j^0 Q_j^0},$$

Domestic intermediate input at basic prices $$M_j = \frac{\sum_i q_i^0 M_{ij}^1}{\sum_i q_i^0 M_{ij}^0}$$

Net product taxes on domestic intermediate inputs $$D_j = \frac{\sum_i d_{ij}^0 q_i^0 M_{ij}^1}{\sum_i d_{ij}^0 q_i^0 M_{ij}^0}$$

Imported intermediate inputs $$M_j^M = \frac{\sum_i q_i^{M0} M_{ij}^{M1}}{\sum_i q_i^{M0} M_{ij}^{M0}}$$

Net product taxes on imported intermediate inputs $$D_j^M = \frac{\sum_i d_{ij}^{M0} q_i^{M0} M_{ij}^{M1}}{\sum_i d_{ij}^{M0} q_i^{M0} M_{ij}^{M0}}$$

Labour input $$L_j = \frac{\sum_l p_{lj}^0 L_{lj}^1}{\sum_l p_{lj}^0 L_{lj}^0}$$

Capital input $$K_j = \frac{\sum_k p_{kj}^0 K_{kj}^1}{\sum_k p_{kj}^0 K_{kj}^0}.$$

The rate industry level TFP growth is then defined, in the usual way, as the difference of the growth rate of the output and that of the weighted average of the inputs, with the value shares of the inputs as weights. After some simplification we get

(48)
$$\Delta t_j = \frac{q_j^0 Q_{0j}^1 - q_j^0 Q_{0j}^0}{q_j^0 Q_j^0} - \frac{\sum_i (q_i^0 M_{ij}^1 - q_i^0 M_{ij}^0)}{q_j^0 Q_j^0} - \frac{\sum_i (d_{ij}^0 q_i^0 M_{ij}^1 - d_{ij}^0 q_i^0 M_{ij}^0)}{q_j^0 Q_j^0}$$
$$- \frac{\sum_i (q_i^{M0} M_{ij}^{M1} - q_i^{M0} M_{ij}^{M0})}{q_j^0 Q_j^0} - \frac{\sum_i (d_{ij}^{M0} q_i^{M0} M_{ij}^{M1} - d_{ij}^{M0} q_i^{M0} M_{ij}^{M0})}{q_j^0 Q_j^0}$$
$$- \frac{\sum_l (p_{lj}^0 L_{lj}^1 - p_{lj}^0 L_{lj}^0)}{q_j^0 Q_j^0} - \frac{\sum_k (p_{kj}^0 K_{kj}^1 - p_{kj}^0 K_{kj}^0)}{q_j^0 Q_j^0},$$

which is a discrete counterpart of the theoretical industry level rate of TFP growth in equation (7). The contributions of intermediate inputs could, and theoretically should, have been written at purchasers' prices (however without trade and transport margins), but to get the impact of taxes and subsidies explicit we are treating them as a separate term.

The Laspeyres quantity indices at the economy level for the deliveries to final demand approach are defined as follows:

Gross output at basic prices
$$Q = \frac{\sum_i q_i^0 Q_i^1}{\sum_i q_i^0 Q_i^0}$$

Intermediate deliveries of domestic products
$$M = \frac{\sum_i q_i^0 M_i^1}{\sum_i q_i^0 M_i^0}$$

Deliveries to final demand of domestic products
$$Y = \frac{\sum_i q_i^0 Y_i^1}{\sum_i q_i^0 Y_i^0}$$

Net product taxes on domestic deliveries to intermediate uses
$$D = \frac{\sum_i d_i^0 q_i^0 M_i^1}{\sum_i d_i^0 q_i^0 M_i^0}$$

Imported intermediate inputs
$$M^M = \frac{\sum_i q_i^{M0} M_i^{M1}}{\sum_i q_i^{M0} M_i^{M0}}$$

Net product taxes on imported intermediate input
$$D^M = \frac{\sum_i d_i^{M0} q_i^{M0} M_i^{M1}}{\sum_i d_i^{M0} q_i^{M0} M_i^{M0}}$$

Labour input $$L = \frac{\sum_l p_l^0 L_l^1}{\sum_l p_l^0 L_l^0}$$

Capital input $$K = \frac{\sum_k p_k^0 K_k^1}{\sum_k p_k^0 K_k^0}.$$

The economy is again treated as a single unit of production.

As can be easily verified the percentage change of deliveries to final demand is obtained as follows:

$$(49) \qquad \Delta Y = \frac{\sum_i q_i^0 Q_i^1 - \sum_i q_i^0 Q_i^0}{\sum_i q_i^0 Y_i^0} - \frac{\sum_i q_i^0 M_i^1 - \sum_i q_i^0 M_i^0}{\sum_i q_i^0 Y_i^0} = \frac{\sum_i q_i^0 Y_i^1 - \sum_i q_i^0 Y_i^0}{\sum_i q_i^0 Y_i^0}.$$

The rate of the economy level TFP growth is then defined as the difference of the growth rate of the deliveries to final demand and the weighted average of the growth rates of the aggregate inputs, with the value shares of the inputs as weights. After some simplification this gives

$$(50) \qquad \begin{aligned} \Delta \overline{T} = {} & \frac{\sum_i q_i^0 Y_i^1 - \sum_i q_i^0 Y_i^0}{\sum_i q_i^0 Y_i^0} - \frac{\sum_i (d_i^0 q_i^0 M_i^1 - d_i^0 q_i^0 M_i^0)}{\sum_i q_i^0 Y_i^0} \\ & - \frac{\sum_i (q_i^{M0} M_i^{M1} - q_i^M M_i^{M0})}{\sum_i q_i^0 Y_i^0} - \frac{\sum_i (d_i^{M0} q_i^{M0} M_i^{M1} - d_i^{M0} q_i^{M0} M_i^{M0})}{\sum_i q_i^0 Y_i^0}, \\ & - \frac{\sum_l (p_l^0 L_l^1 - p_l^0 L_l^0)}{\sum_i q_i^0 Y_i^0} - \frac{\sum_k (p_k^0 K_{kj}^1 - p_k^0 K_k^0)}{\sum_i q_i^0 Y_i^0} \end{aligned}$$

which is a discrete version of the theoretical economy level rate of TFP growth in equation (25).

Domar-aggregation rule is obtained the familiar way by multiplying the industry level rates in equation (48) by the ratio of industry gross output to the economy level deliveries to final demand, adding up over industries and subtracting the result from both sides of equation (50). After some manipulation this leads to

$$\Delta T = \sum_j \frac{q_j^0 Q_j^0}{\sum_i q_i^0 Y_i^0} \Delta t_j$$

$$+ \frac{\sum_j \sum_i (d_{ij}^0 q_i^0 M_{ij}^1 - d_{ij}^0 q_i^0 M_{ij}^0)}{\sum_i q_i^0 Y_i^0} - \frac{\sum_i (d_i^0 q_i^0 M_i^1 - d_i^0 q_i^0 M_i^0)}{\sum_i q_i^0 Y_i^0}$$

$$+ \frac{\sum_j \sum_i (d_{ij}^{M0} q_i^{M0} M_{ij}^{M1} - d_{ij}^{M0} q_i^{M0} M_{ij}^{M0})}{\sum_i q_i^0 Y_i^0} - \frac{\sum_i (d_i^{M0} q_i^{M0} M_i^{M1} - d_i^{M0} q_i^{M0} M_i^{M0})}{\sum_i q_i^0 Y_i^0}$$

$$+ \frac{\sum_j \sum_l (p_{lj}^0 L_{lj}^1 - p_{lj}^0 L_{lj}^0)}{\sum_i q_i^0 Y_i^0} - \frac{\sum_l (p_l^0 L_l^1 - p_l^0 L_l^0)}{\sum_i q_i^0 Y_i^0}$$

$$+ \frac{\sum_j \sum_k (p_{kj}^0 K_{kj}^1 - p_{kj}^0 K_{kj}^0)}{\sum_i q_i^0 Y_i^0} - \frac{\sum_k (p_k^0 K_k^1 - p_k^0 K_k^0)}{\sum_i q_i^0 Y_i^0}. \tag{51}$$

Equation (51) is a discrete counterpart of equation (35). The second and third rows of its right hand side disappear if the tax rates are identical in all intermediate uses of each of the domestic products as well in all intermediate uses of each of the imported products.

In the value added approach the growth rate of the industry level TFP is then defined as the difference of the growth rate of the industry's value added and that of the weighted average of the growth rates of aggregate labour and capital inputs, with the value shares of the inputs as weights.

$$\Delta t_j^v = \frac{v_j^0 V_j^1 - v_j^0 V_j^0}{v_j^0 V_j^0} - \frac{\sum_l (p_{lj}^0 L_{lj}^1 - p_{lj}^0 L_{lj}^0)}{q_j^0 V_j^0} - \frac{\sum_k (p_{kj}^0 K_{kj}^1 - p_{kj}^0 K_{kj}^0)}{q_j^0 V_j^0}, \tag{52}$$

which is a discrete counterpart of the theoretical equation (11). Since

$$\Delta V_j = \frac{v_j^0 V_j^1 - v_j^0 V_j^0}{v_j^0 V_j^0}$$

(53)
$$= \frac{q_j^0 Q_j^1 - \sum_i q_i^0 M_{ij}^1 - \sum_i d_{ij}^0 q_i^0 M_{ij}^1 - \sum_i q_i^{M0} M_{ij}^{M1} - \sum_i d_{ij}^{M0} q_i^{M0} M_{ij}^{M1}}{v_j^0 V_j^0}$$

$$- \frac{q_j^0 Q_j^0 - \sum_i q_i^0 M_{ij}^0 - \sum_i d_{ij}^0 q_i^0 M_{ij}^0 - \sum_i q_i^{M0} M_{ij}^{M0} - \sum_i d_{ij}^{M0} q_i^{M0} M_{ij}^{M0}}{v_j^0 V_j^0}$$

we, in view of equation (48), have

(54)
$$\Delta t_j^v == \frac{q_j^0 Q_j^0}{v_j^0 V_j^0} \Delta t_j ,$$

which again is a discrete counterpart of equation (17).

The rate of economy level TFP growth is, in the approach based on the production possibilities frontier, defined, in accordance with theoretical rate of growth in equation (29), as follows:

(55)
$$\Delta T^v = \frac{\sum_j v_j^0 V_j^1 - \sum_j v_j^0 V_j^0}{\sum_j v_j^0 V_j^0} - \frac{\sum_l (p_l^0 L_l^1 - p_l^0 L_l^0)}{\sum_j v_j^0 V_j^0} - \frac{\sum_k (p_k^0 K_{kj}^1 - p_k^0 K_k^0)}{\sum_j v_j^0 V_j^0} .$$

Multiplying both sides of equation (52) by the ratio of industry's value added to the sum of the value added of all the industries and subtracting the result from both sides of equation (55) gives

(56)
$$\Delta T^v = \sum_j \frac{v_j^0 V_{ji}^0}{\sum_j v_j^0 V_j^0} \Delta t_j^v$$
$$+ \left(\frac{\sum_j \sum_l (p_{lj}^0 L_{lj}^1 - p_{lj}^0 L_{lj}^0)}{\sum_j v_j^0 V_j^0} - \frac{\sum_l (p_l^0 L_l^1 - p_l^0 L_l^0)}{\sum_j v_j^0 V_j^0} \right) ,$$
$$+ \left(\frac{\sum_j \sum_k (p_{kj}^0 K_{kj}^1 - p_{kj}^0 K_{kj}^0)}{\sum_j v_j^0 V_j^0} - \frac{\sum_k (p_k^0 K_k^1 - p_k^0 K_k^0)}{\sum_j v_j^0 V_j^0} \right)$$

which is a discrete version of the theoretical equation (37).

The rate of economy level rate of TFP growth based on the value added of the economy as a whole can be calculated in the line of equation (42). The aggregation rule from the industry level is obtained from equation (46). In our empirical application we do not however assume that there is only one homogeneous product for the group of industries represented in this case by the entire economy but do assume that all the industries pay identical prices for their inputs, intermediate inputs included.

Remark 9. We have been able to derive a system based on Laspeyres indices, which is in accordance with our theoretical formulas and confirms our theoretical results. All the indices of the theoretical part have not been covered, but by now it is obvious that the results would be analogous to those obtained in it. This of course follows from the fact of the Laspeyres indices being additive and as such consistent in aggregation. [8]

7. Application Based on Törnqvist indexes

The logarithms of industry-level Törnqvist quantity indices are defined as follows:

Output at basic prices $\Delta \log Q_j = \log q_j^0 Q_j^1 - \log q_j^0 Q_j^0$,

Deliveries to final demand at basic prices $\Delta \log Y_j = \log q_j^0 Y_j^1 - \log q_j^0 Y_j^0$,

Average value share $\bar{u}_i = \overline{\frac{u_i}{\sum_i u_i}} = (\frac{u_i^0}{\sum_i u_i^0} + \frac{u_i^1}{\sum_i u_i^1})\, 1/2$,

Domestic intermediate inputs at purchasers' prices[9] $\Delta \log M_j = \sum_i \overline{\frac{(1+d_{ij})q_i M_{ij}}{\sum_i (1+d_{ij})q_i M_{ij}}} \Delta \log M_{ij}$,

Domestic intermediate inputs at basic prices $\Delta \log MB_j = \sum_i \overline{\frac{q_i M_{ij}}{\sum_i q_i M_{ij}}} \Delta \log M_{ij}$,

where $\Delta \log M_{ij} = \log q_i^0 M_{ij}^1 - \log q_i^0 M_{ij}^0$. [10]

---

[8] Vartia (1976) defines an index number formula to be consistent in aggregation if the value of the index calculated in two stages necessarily coincides with the value of the index calculated in and ordinary way, i.e. in a single stage. According to Diewert(1978) superlative index numbers are approximately consistent in aggregation.

[9] Excluding trade and transport margins.

Imported intermediate inputs at purchasers' prices $\Delta\log M_j^M = \sum_i \frac{\overline{(1+d_{ij}^M)q_i^M M_{ij}^M}}{\sum_i (1+d_{ij}^M)q_i^M M_{ij}^M}\Delta\log M_{ij}^M$,

Imported intermediate inputs at basic prices $\Delta\log MB_j^M = \sum_i \frac{\overline{q_i^M M_{ij}^M}}{\sum_i q_i^M M_{ij}^M}\Delta\log M_{ij}^M$

where $\Delta\log M_{ij}^M = \log q_i^0 M_{ij}^{M1} - \log q_i^0 M_{ij}^{M0}$.

Labour input $\Delta\log L_j = \sum_l (\frac{\overline{p_{lj}L_{lj}}}{\sum_l p_{lj}L_{lj}}\Delta\log L_{lj})$

Capital input $\Delta\log K_j = \sum_k (\frac{\overline{p_{kj}K_{kj}}}{\sum_k p_{kj}K_{kj}}\Delta\log K_{kj})$.

We use the following notation for value shares:

$$\overline{u}_j^{MQ} = \frac{\overline{\sum_i (1+d_{ij})q_i M_{ij}}}{q_j Q_j},\ \overline{u}_j^{MBQ} = \frac{\overline{\sum_i q_i M_{ij}}}{q_j Q_j},\ \overline{u}_j^{MMQ} = \frac{\overline{\sum_i (1+d_{ij}^M)q_i^M M_{ij}^M}}{q_j Q_j},$$

$$\overline{u}_j^{MMBQ} = \frac{\overline{\sum_i q_i^M M_{ij}^M}}{q_j Q_j}$$

$$\overline{u}_j^{LQ} = \frac{\overline{\sum_l p_{lj}L_{lj}}}{q_j Q_j},\ \overline{u}_j^{KQ} = \frac{\overline{\sum_k p_{kj}K_{ij}}}{q_j Q_j},\ \overline{u}_j^{VQ} = \frac{v_j V_j}{q_j Q_j}.$$

The rate of industry level TFP-change is now defined as follows:

$$\begin{aligned} &\Delta\log t_j = \Delta\log Q_j - \overline{u}_j^{MBQ}\Delta\log MB_j - (\overline{u}_j^{MQ}\Delta\log M_j - \overline{u}_j^{MBQ}\Delta\log MB_j) \\ (57)\quad &-\overline{u}_j^{MMBQ}\Delta\log MB_j^M - (\overline{u}_j^{MMQ}\Delta\log M_j^M - \overline{u}_j^{MMBQ}\Delta\log MB_j^M) \\ &-\overline{u}_j^{LQ}\Delta\log L_j - \overline{u}_j^{MQ}\Delta\log K_j \quad . \end{aligned}$$

The lack of consistency in aggregation makes the empirical application based on the Törnqvist indices more intricate than the one based on the Laspeyres indices. It is not possible to directly break down the rate of change of the intermediate inputs into the contributions of the intermediate inputs at basic prices on the one hand and of the tax margins on the other. Therefore the terms on the RHS of equation (57) are needed to catch the impact of the taxes on products.

[10] We assume volume changes in the elementary indices at basic prices and purchaser's prices to be identical, i.e. we assume the volume of the product tax to change at the same rate as the input on which it is paid.

For the economy level we define the following logarithms of Törnqvist quantity indices:

Gross output at basic prices $$\Delta \log Q = \sum_i \overline{\frac{q_i Q_i}{\sum_i q_i Q_i}} \Delta \log Q_i$$

Intermediate uses of domestic products at purchasers' prices $$\Delta \log M = \sum_i \overline{\frac{(1+d_i) q_i M_i}{\sum_i (1+d_i) q_i M_i}} \Delta \log M_i$$

Intermediate uses of domestic products at BP $$\Delta \log MB = \sum_i \overline{\frac{q_i M_i}{\sum_i q_i M_i}} \Delta \log M_i ,$$

where $\Delta \log M_i = \log q_i^0 M_i^1 - \log q_i^0 M_i^0$.

Imported intermediate inputs at PP

$$\Delta \log M^M = \sum_i (\overline{\frac{(1+d_i^M) q_i^M M_i^M}{\sum_i (1+d_i^M) q_i^M M_i^M}} \Delta \log M_i^M),$$

Imported intermediate inputs at BP $$\Delta \log MB^M = \sum_i (\overline{\frac{q_i M_i^M}{\sum_i q_i M_i^M}} \Delta \log M_i^M),$$

where $\Delta \log M_i^M = \log q_i^0 M_i^{M1} - \log q_i^0 M_i^{M0}$.

Labour input $$\Delta \log L = \sum_l (\overline{\frac{p_l L_l}{\sum_l p_l L_l}} \Delta \log L_l)$$

Capital input $$\Delta \log K = \sum_k (\overline{\frac{p_k K_k}{\sum_k p_k K_k}} \Delta \log K_k).$$

We use the following notation for value shares:

$$\overline{u}_j^Q = \overline{\frac{q_j Q_j}{\sum_j q_j Q_j}},\ \overline{u}_j^V = \overline{\frac{v_j V_j}{\sum_j v_j V_j}},\ \overline{u}_j^Y = \overline{\frac{q_j Y_j}{\sum_j q_j Y_j}},\ \overline{u}^{MQ} = \overline{\frac{\sum_i (1+d_i) q_i M_i}{\sum_i q_i Q_i}},\ \overline{u}^{MBQ} = \overline{\frac{\sum_i q_i M_i}{\sum_i q_i Q_i}},$$

$$\overline{u}^{MMQ} = \overline{\frac{\sum_i (1+d_i^M) q_i^M M_i^M}{\sum_i q_i Q_i}},\ \overline{u}^{MMBQ} = \overline{\frac{\sum_i q_i^M M_i^M}{\sum_i q_i Q_i}},\ \overline{u}^{LQ} = \overline{\frac{\sum_l p_l L_l}{\sum_i q_i Q_i}},$$

$$\overline{u}^{KQ} = \overline{\frac{\sum_k p_k K_k}{\sum_i q_i Q_i}},\ \overline{u}^{YQ} = \overline{\frac{\sum_i q_i Y_i}{\sum_i q_i Q_i}},\ \overline{u}^{VQ} = \overline{\frac{\sum_j v_j V_j}{\sum_j q_j Q_j}}.$$

The rate of economy level TFP growth based on gross output obviously can be written as follows:

(58)
$$\begin{aligned}\Delta\log T = \Delta\log Q - \bar{u}^{MBQ}\Delta\log MB - (\bar{u}^{MQ}\Delta\log M - \bar{u}^{MBQ}\Delta\log MB)\\ - \bar{u}^{MMBQ}\Delta\log MB^{M} - (\bar{u}^{MMQ}\Delta\log M^{M} - \bar{u}^{MMBQ}\Delta\log MB^{M})\\ - \bar{u}^{LQ}\Delta\log L - \bar{u}^{KQ}\Delta\log K.\end{aligned}$$

Again the impact of the taxes on products is caught by the two terms in parenthesis on the RHS of equation (58).

A double deflated measure for the deliveries to final demand at basic prices is obtained as the difference of the growth rate of output and that of intermediate inputs valued at basic prices:

(59)
$$\Delta\log Y_1 = (\bar{u}^{YQ})^{-1}\left(\Delta\log Q - \bar{u}^{MBQ}\Delta\log MB\right).$$

Multiplying both sides of equation (58) by share of deliveries of domestic output to final demand gives in total output yields, in view of equation (59), the following expression of the rate of economy level TFP change based on the double deflated deliveries to final demand:

(60)
$$\begin{aligned}\Delta\log \overline{T_1} = \Delta\log Y_1 - (\bar{u}^{YQ})^{-1}\left(\bar{u}^{MQ}\Delta\log M - \bar{u}^{MBQ}\Delta\log MB\right)\\ - (\bar{u}^{YQ})^{-1}\bar{u}^{MMBQ}\Delta\log MB^{M} + (\bar{u}^{YQ})^{-1}(\bar{u}^{MMQ}\Delta\log M^{M} - \bar{u}^{MMBQ}\Delta\log MB^{M})\\ - (\bar{u}^{YQ})^{-1}\bar{u}^{LQ}\Delta\log L - (\bar{u}^{YQ})^{-1}\bar{u}^{KQ}\Delta\log K) \quad .\end{aligned}$$

The impact of the taxes on products are now represented by the second and forth terms on the RHS.

For Domar-aggregation both sides of equation (57) for each industry are multiplied by the following aggregation coefficients:

(61)
$$C_j = \left(\bar{u}^{YQ}\right)^{-1}\bar{u}_j^{Q}$$

Summing up across industries and subtracting the result from both sides of equation (60) for the economy level produces, in view of equation (59), the following aggregation rule equation from the industry level to the economy level:

$$
\begin{aligned}
\Delta \log \overline{T}_1 = {} & \left(\overline{u}^{YQ}\right)^{-1}[\sum_j \overline{u}_j^Q \Delta \log t_j \\
& + (\sum_j \overline{u}_j^Q \overline{u}_j^{MQ} \Delta \log M_j - \overline{u}^{MQ} \Delta \log M) \\
& + (\sum_j \overline{u}_j^Q \overline{u}_j^{MMQ} \Delta \log M_j^M - \overline{u}^{MMQ} \Delta \log M^M) \\
& + (\sum_j \overline{u}_j^Q \overline{u}_j^{LQ} \Delta \log L_j - \overline{u}^{LQ} \Delta \log L) \\
& + (\sum_j \overline{u}_j^Q \overline{u}_j^{KQ} \Delta \log K_j - \overline{u}^{KQ} \Delta \log K)]
\end{aligned} \tag{62}
$$

Rows 2-5 disappear if all the rates of growth of respective inputs as well as their shares in total value of output are identical across industries. Identical prices do not make the reallocation terms to disappear in the case of the Törnqvist indices.

Another possible measure of the growth rate of the deliveries to final demand is the single deflated growth rate:

$$
\Delta \log Y = \sum_i \overline{u}_i^Y \Delta \log Y_i . \tag{63}
$$

The difference between the two measures can be written as follows:

$$
\Delta \log Y - \Delta \log Y_1 = \sum_i \overline{u}_i^Y \Delta \log Y_i - (\overline{u}^{YQ})^{-1}\left(\Delta \log Q - \overline{u}^{MBQ} \Delta \log MB\right). \tag{64}
$$

Obviously the difference disappears only if the growth rates of output and intermediate inputs are identical and the shares of intermediate deliveries in total output are identical in the two years of comparison.

Furthermore from equations (60) and (64) we get the economy level measure of the rate of TFP growth based on the single deflated deliveries to final demand as follows:

$$
\Delta \log \overline{T} = \Delta \log \overline{T}_1 + \sum_i \overline{u}_i^Y \Delta \log Y_i - (\overline{u}^{YQ})^{-1}\left(\Delta \log Q - \overline{u}^{MBQ} \Delta \log MB\right). \tag{65}
$$

Remark 11. The Törnqvsit index for the economy level rate of TFP growth based on the double deflated deliveries to final demand (equation 62) depends besides the rates of industry level rates also on reallocation of inputs across industries. Reallocation term relating to a category of input disappears only if the growth rate of the inputs as well as their value shares in industry's total output are identical across industries. The index based on the single deflated deliveries to final demand depends also on the reallocation of industry outputs between deliveries to final demand and to intermediate uses (equation 65).

The double deflated rate of growth of the industry level value-added is defined as follows:

$$(66) \qquad \Delta \log V_j = \left(\overline{u}_j^{VQ}\right)^{-1} \Delta \log Q_j - \left(\overline{u}_j^{VQ}\right)^{-1} \overline{u}_j^{MQ} \Delta \log M_j - \left(\overline{u}_j^{VQ}\right)^{-1} \overline{u}_j^{MMQ} \Delta \log M_j^M .$$

For the industry-level rate of TFP-change we thus get:

$$(67) \qquad \Delta \log t_j^v = \Delta \log V_j - \left(\overline{u}_j^{VQ}\right)^{-1} \overline{u}_j^{LQ} d \log L_j - \left(\overline{u}_j^{VQ}\right)^{-1} \overline{u}_j^{KQ} d \log K_j .$$

This together with equations (57) and (66) yields the familiar relationship between the value added based and gross output based industry level measures:

$$(68) \qquad \Delta \log t_j^v = \left(\overline{u}_j^{VQ}\right)^{-1} \Delta \log t_j .$$

The economy level rate of value added growth, based on the production possibilities frontier, is defined as the weighted average of the industry level rates:

$$(69) \qquad \Delta \log V = \sum_j \overline{u}_j^V \Delta \log V_j$$

The economy level rate of TFP growth is defined analogously to the industry level as follows:

$$(70) \qquad \Delta \log T^v = \Delta \log V - \left(\overline{u}^{VQ}\right)^{-1} \overline{u}^{LQ} \Delta \log L - \left(\overline{u}^{VQ}\right)^{-1} \overline{u}^{KQ} \Delta \log K .$$

Multiplying both sides of the equation (67) by $\overline{u}_j^V$, adding up across industries and subtracting the result from both sides of (70) gives:

$$
\Delta \log T^{v} = \sum_{j} \bar{u}_{j}^{V} \Delta \log t_{j}^{v}
$$

(71) $$
+ \left[\sum_{j} \bar{u}_{j}^{V} \left(\bar{u}_{j}^{VQ} \right)^{-1} \bar{u}_{j}^{LQ} \Delta \log L_{j} - \left(\bar{u}^{VQ} \right)^{-1} \bar{u}^{LQ} \Delta \log L \right] .
$$

$$
+ \left[\sum_{j} \bar{u}_{j}^{V} \left(\bar{u}_{j}^{VQ} \right)^{-1} \bar{u}_{j}^{KQ} \Delta \log K_{j} - \left(\bar{u}^{VQ} \right)^{-1} \bar{u}^{KQ} \Delta \log K \right]
$$

Remark 12. The Törnqvsit index for the economy level rate of TFP growth based on the production possibilities frontier of the industries' value added depends besides the industry level rates also on the reallocation of labour and capital inputs between industries. These reallocation terms disappear only if the rates of growth of the respective inputs as well as their value shares in industry's total output are identical across industries (equation 71).

If the economy is treated as single unit of production with identical prices paid by all of the industries for all the inputs, however producing a number of different products, the term representing the reallocation of value added has to be included in the aggregation rule in line with equation (46). The rate of growth of the economy level value added is now defined in line with first expression in equation (40).

8. Calculations based on the Finnish data

The theoretical results were tested by an empirical experiment with the Finnish data for 1999 and 2000. There are several choices to be made in an empirical application. Each of them is likely to have an effect on the results of the calculations. *The first choice* is whether to use supply and use tables or symmetric input-output tables. We were using SIOTs since all our formulas are based on them.[11].

The second choice concerns the level of aggregation in the calculations as well as the level of aggregation in the deflation. E.g. the draft Eurostat Input-Output manual suggests the deflation to be performed at the lowest possible level. Our empirical exercise is based on the Finnish supply

[11] The problem in adapting the formulas for supply and use tables is of course the fact an industry can produce several products and a product can be produced by several industries. The formulas based on production possibilities frontier for industry level value added could be adapted in line with Jorgenson, Ho and Stiroh (2004) by defining a uniform price for the output of each industry as well as the calculating the price index for each product using the industry shares as weights. The assumptions on which these calculations are based are of course similar to those used in the derivation of SIOTs from SUTs. Adapting the formulas for final demand approach would be more difficult, since it would be necessary to keep the breakdown of the indutstries' output by product.

and use tables for 1999 and 2000, at current and fixed prices, with about 950 different products and about 180 industries. Deflation was, for the fixed price tables, originally performed, and the tables balanced, at this detailed level. But if deflation is performed at a more detailed level than the actual calculations then the basic price at a more aggregate level (i.e. its rate of change) can be different in different uses, because the aggregate level products in different uses consist of different baskets of the more detailed level products. Therefore we redeflated the tables at the level of 55 industries used in the calculation of the productivity measures.

The third choice concerns the price concept on which the deflator is based. E.g. in JGF (1987) outputs are valued at basic prices, to use the terminology of the SNA93, and inputs are valued at purchasers' prices without trade and transport margins, in other words at basic prices plus taxes, net of similar subsidies, on products. Each industry's output is deflated by its output deflator at basic price. The intermediate deliveries from an industry are deflated by an output deflator that includes the net taxes on products paid for that output. Including taxes net of subsidies on products in a deflator would require that the tax rate is the same in all uses of that product. In the case of the value added tax this is not normally true. Even in the case of the rest of the product taxes/subsidies there can be problems, since the product baskets in different uses consist of different detailed level products with, possibly, different tax rates. To be able to test our theoretical formulas we ended up deflating all the uses of the domestic output of an aggregate level product by its implicit output deflator at basic price. Respectively all the uses of an aggregated level imported product were deflated by its average deflator at basic (c.i.f.) price in all its uses. The growth rates of product taxes on intermediate inputs were assumed to be equal to the growth rates of respective inputs.

The fourth choice concerns the index number formula. We made the calculations both using the, additive Laspeyres indices and also using the, nonadditive but superlative, Törnqvist indices. The differences between the applications based on these two types of indices were discussed above.

Since the main purpose of our empirical exercise was to test our theoretical results concerning the different approaches of aggregating the industry level measures to the economy level, we did not make an effort to allocate our labour input to different categories. The labour compensation of the self-employed was estimated on the basis of the hourly compensation of the employees in each of the industries. The estimates depend, as of course could be expected, on the level of aggregation of the industries. Likewise the different categories of fixes capital were suppressed in our calculations. Gross return on capital was calculated by subtracting from the operating surplus the estimated labour compensation of the self-employed.

The results of the calculations are reported in Tables 1 and 2. The calculations based on single deflated deliveries to final demand were performed using both of the index number formulas. Likewise, the calculations based on the production possibilities frontier of the industries' value added (Value added, IL) as well as those based on the assumption of the existence of the economy level value added function (Value added, EL) were performed with both of the index number formulas. For Törnqvist indicices also calculations based on double deflated deliveries to final demand (Final demand, DD) were performed. From the results (Table 1) it is obvious that both the choice of the measure of output as well as of the index number formula matter in the estimation of the economy level rate of TFP growth. In the case of the Laspeyres indices the results range from 1,9 to 2,4 and those based on Törnqvist indices from 2,0 to 3,2. These differences are caused by the differences in definitions of the output measures.

The contributions of industry-level TFP growth to the economy level TFP growth vary from 1,3 to 1,6 in the case of Laspeyres indices and the 1,9 to 2,5 in the case of Törnqvist indices. Reallocation of domestically produced products in intermediate uses reduces significantly the economy level TFP growth when measured by Törnqvist indices. The reallocation of imported product in intermediate uses contributes to some extent to the overall TFP-growth in this case. Reallocation of labour and capital contribute significantly in all cases. In the case of the Törnqvist indices based on the economy level value added this contribution is loosed in the reallocation of value added.

9. Concluding remarks

Our objective was to compare different approaches to the aggregation of industry level TFP measures to the economy level. Special attention was paid to the implications of the existence of taxes and subsidies on products. Both at the industry level and at the economy level different measures of output growth were considered. The relationship of the two industry level TFP measures, the one based on gross value and the one based on the double deflated value added, is simply determined by the share of the value added in gross output. This was true both of the measures based on Laspeyres indices as well as of those based on Törnqvist indices.

At the economy level four different measures of output were distinguished: 1) deliveries to final demand based on single deflation, 2) deliveries to final demand based on double deflation, 3) value added based on the production possibilities frontier of the industries' value added and 4) value added based on the production function of the entire economy. In the case of the Laspeyres indices the second one is not relevant and the two last ones, in our empirical experiment, differ only slightly. This difference is caused by the impact of the reallocation of value added relating to net taxes on products in intermediate uses. The ratio between the measures 1 and 3 is the inverse of the ratio of the value of deliveries to final demand to the value added.

In the case of Törnqvist indices all the four different measures are relevant. The difference between the two first measures consists of the reallocation of deliveries to final demand by industries.

The difference between the two last measures is caused by the reallocation of value added. The ratio of the measure based on the single deflated deliveries to final demand to the measure based on the economy level value added is again the inverse of the ratio of the respective shares in gross output. In this case the shares of course are calculated as averages of the shares in the years of comparison.

The contributions of the industry level rates of TFP growth are, for the Laspeyres indices, in inverse proportion to the values of respective economy level measures of output in the basic year. This goes also for the contributions of the reallocation of labour and capital inputs. In the case of the Törnqvist indices this inverse relationships are not exact, since ratio of output measures has to be calculated as the ratio of the average shares of value added and that of deliveries to final demand in the gross output.

The contributions of the reallocation of domestically produced and imported products in intermediate uses seem to be rather insignificant in the case of the Laspeyres indices. This is due to the fact that basic prices were assumed to be identical in all uses. In the case of the Laspeyres indices identical prices make reallocation factor to disappear. It is only the different tax rates in different uses that can cause this reallocation term to be nonzero. For the Törnqvist indices identical prices do not make the reallocation term to disappear. Accordingly, in our empirical experiment, the contributions of reallocation of both domestically produced and imported products in intermediate uses are more substantial. But they are only partly due to the differences in the tax rates.

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Table 1a. Economy level TFP-growth and contributions to the economy level growth of output, Laspeyres indices, per cent

	Measure of output		
	Deliveries to FD	Value added, IL	Value added, EL
1 Growth of output	6,343	3,846	3,853
2 Taxes on domestic products in intermediate uses	- 0,063		
3 Subsidies on domestic products in intermediate uses	0,086		
4 Net taxes on domestic products in intermediate uses (2-3)	- 0,149		
5 Imported intermediate inputs	3,435		
6 Net taxes on imported prodócts in intermediate uses	0,002		
7 Labour	0,682	0,860	0,860
8 Capital	0,484	0,610	0,610
9 Economy level TFP growth (1-4-5-6-7-8)	1,890	2,376	2,383

Table 1b. Economy level TFP growth and contributions to the economy level growth of output, Törnqvist indices, per cent

	Measure of output			
	Deliveries to FD	Deliveries to FD, DD	Value added, IL	Value added, EL
1 Growth of output	6,233	6,140	4,700	4,002
2 Domestic products in intermediate uses at pp	4,276	4,276		
3 Domestic products in intermediate uses at bp	4,515	4,515		
4 Net taxes on domestic prodcts in intermediate uses (2-3)	-0,239	-0,239		
5 Imported intermediate inputs at pp	3,256	3,256		
6 Imported intermediate inputs bp	3,256	3,422		
7 Net taxes on imported intermediate inputs	0,000	-0,166		
8 Labour	0,661	0,661	0,847	0,847
9 Capital	0,482	0,482	0,617	0,617
10 Economy level TFP growth (1-4-5-8-9)	2,074	1,981	3,236	2,538

Table 2a. Contributions to the economy level TFP growth, Laspeyres indices, per cent

	Measure of output		
	Deliveries to FD	Value added, IL	Value added, EL
1 Industry level TFP growth	1,293	1,630	1,630
2 Reallocation of (3+4+5+6+7)	0,597	0,746	0,753
3 - Value added			0,007
4 - Intermediate uses of domestic products	-0,050		
5 - Intermediate uses of imported products	0,056		
6 - Labour	0,279	0,352	0,352
7 - Capital	0,313	0,394	0,394
8 Economy level rate of TFP growth (1+2)	1,890	2,376	2,383

Table 2b. Contributions to the economy level TFP growth, Törnqvist indices, per cent

	Measure of output			
	Deliveries to FD	Deliveries to FD,DD	Value added, IL	Value added, EL
1 Industry level TFP growth	1,891	1,891	2,527	2,527
2 Reallocation of (3+4+5+6+7)	0,182	0,090	0,708	0,010
3 - Final demand / Value added	0,093			-0,698
4 - Intermediate uses of domestic products, pp	-0,580	-0,580		
(intermediate uses of domestic products, bp)	-0,460	-0,460		
5 - Intermediate uses of imported products, pp	0,110	0,110		
(intermediate uses of i ported products, bp)	0,117	0,117		
6 - Labour	0,265	0,265	0,332	0,332
7 - Capital	0,295	0,295	0,376	0,376
8 Economy level rate of TFP growth (1+2)	2,074	1,981	3,236	2,538

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