

# **Economics and the Price Index**

**S.N. Afriat  
Carlo Milana**

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## Economics and the Price Index

A theft amounting to £1 was a capital offence in 1260, and a judge in 1610 affirmed the law could not then be applied since £1 was no longer what it was. Such association of money with a date is well recognized for its importance in very many different connections. Thus arises the need to know how to convert an amount at one date into an equivalent amount at another date. In other words, a *price index*. The longstanding question concerning how such an index should be constructed is known as 'The Index Number Problem'.

A pervasive long established institution for economics, it is a number issued by the Statistical Office that should tell anyone the ratio of costs of maintaining a given standard of living in two periods where prices differ. For a succession of three periods, the product of the ratios for successive pairs must coincide with the ratio for the endpoints. This is the *chain consistency* required of price indices. A usual supposition is that the index is determined by an algebraical formula involving price and quantity data for the two reference periods, as with the one or two hundred formulae in the collection of Irving Fisher, always joined with the question of which one to choose, and the perplexity that chain consistency is not obtained with any. Hence finally they should all be abandoned. This is the reality of 'The Index Number Problem'.

Now in this book consistent prices indices for any number of periods are all computed together to make a resolution of the 'Problem', proved *unique* hence never to be joined by others to make a Fisher-like proliferation. That brings to a conclusion an issue giving rise to extensive thought and theory to which over the decades a remarkable number of economists have each contributed a word, or volume.

A product of attention to the 'Problem' over a half-century, this book should be of interest to all those for whom 'The Index Number Problem' remains, beside of perpetual practical concern, a source of fascination.

**S. N. Afriat**, resident of Siena, intermittent adjunct at the University, Mathematics and Economics, permanent Visiting Professor.

**Carlo Milana**, Research Director at ISAE (Istituto di Studi e Analisi Economica), Rome, a public research institution supported by the Italian Ministry of Economy and Finance.

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**A** friat is the guru of the price index ... his work is a classic illustration of how much we learn from new ways of thinking.

**Angus Deaton**

Dwight D. Eisenhower Professor of International Affairs  
Professor of Economics  
Princeton University, USA

**R**esearchers interested in price indexes, inter-area comparisons, revealed preference theory, or productivity measurement will find this book an enlightening, entertaining, and highly creative treatise on where the theory should go from here.

**Marshall Reinsdorf**

Senior Research Economist  
US Bureau of Economic Analysis

**S**ydney Afriat is famous for his unique and penetrating insights, often very unsettling to those who have worked long and hard in a field—without ever seeing what is obvious to Sydney, who then formulates it neatly in very compact mathematics. In this case he has a very good co-author who has independently developed a critical examination of many current writings on the subject, and has helped him develop his insights in practical directions. This monograph will very likely render many current discussions of index numbers obsolete.

**Edward Nell**

Malcolm B. Smith Professor of Economics  
New School University, NY

**N**ew generation price index workers can have pleasure to escape stagnation in the Irving Fisher trap and catch on to the new approach with its conceptual fidelity and computational elegance.

**Preface**

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**Foreword**

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Sydney Afriat is the guru of the price index. As a young mathematician, he arrived at Richard Stone's Department of Applied Economics in Cambridge in the early 1950s, then the great center of research on theoretical and applied consumer behaviour. He soon realized that neither he nor anyone else knew very much about what was meant by "the price index", in spite of being part of the everyday discourse of economics. In the half century since then, he has been exploring the foundations of the topic. Over the years, he has produced beautiful theorems on the topic, many of them completely unexpected even by the *cognoscenti* of the topic. And where he has led, the profession has followed, often many years later. Along the way, his work has fathered important incidental areas in economics, perhaps most notably the non-parametric analysis of demand, that has carried into other areas, like the analysis of efficiency with his frontier production function that extends Farrell's method and the stochastic frontier.

Afriat is one of those rare and rarely gifted individuals who think differently from everyone else. What is obvious to him can sometimes seem bizarre to others, especially at first, and his vision of "the price index" often differs sharply from those that dominate the profession. Yet his work is a classic illustration of how much we learn from new ways of thinking, and how the bizarre and unfamiliar conceptions and results of an idiosyncratic leader can become the orthodoxy of the next generation.

The last book presented a coherent discussion of fifty years of astonishingly creative work on the price index. Some of the analysis had appeared before but much had not. Afriat's many friends may have welcomed the definitive account, as also those who had not previously had the opportunity to understand and enjoy the work.

So much is said in my foreword to that book. Now comes the surprise, that the story is not there ended. At the heart of this new book is a paper of twenty-five years ago in a volume in honour of Sir Richard Stone which I edited, reproduced in Part II. Yes very original, quite unlike anything seen before, and then what? Read on.

**Angus Deaton**

Dwight D. Eisenhower Professor of International Affairs  
Professor of Economics  
*Princeton University, USA*

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# Preface

Though the mathematics of the method, its theoretical rationalization and computations, require an account, the scheme for applications is simple, and conveys an idea of what could be meant by an answer to “The Index Number Problem”.

A price index formula based on a pair of reference periods has conventionally been algebraical and involved data for those periods alone. Then there are inconsistencies between formulae in the treatment of more than two periods, conflicting with the nature of price indices as such, as gathered by Irving Fisher’s “Tests”.

Formulae proposed now are of an entirely different type and are computed simultaneously for any number of periods, involving the data for all of them, without any of the multi-period consistency problems that go with the conventional formulae. There is either exactness, subject to a condition on the data, or approximation, for the hypothetical underlying utility which in any case there is no need to actually construct.

The theory of the price index, proper, starts with the Utility Cost Factorization Theorem going back to 1950’s. By itself it represents no resolution of the Index Number Problem, nor had there even been a real idea of what could be meant by such a resolution.

The method now proposed does convey some idea of what could be meant by such a resolution. It even represents such a resolution itself.

The method has been available in the main for more than twenty five years, apart from amplifications made just now. But only recently has it been recognized as a proper resolution of the Index Number Problem. The here provided first exercises with the arithmetic go to convey the practicality of it.

The needs of dealing with the EUKLEMS Project data when it became available, joined with chance encounter with data of use for practising the new computations and development of needed software, have stirred into life that almost forgotten work.

Here provided in Part I are three papers by present authors:

“The Super Price Index: Irving Fisher, and after”

has more to do with history.

“The Price Level Computation Method”

is an exposition of the mathematics, and.

“Price Level Computation: Illustrations”

offers first exercises with the arithmetic of the method that go to convey the practicality of it.

It is submitted that, for the very large number of different traditional type formulae to determine *price indices* associated with a pair of periods, which are joined with the long-standing question of which one to choose, they should all be abandoned. For the method proposed instead, *price levels* associated with periods are first all computed together, subject to a consistency of the data, and then price indices that are all true taken together are determined from their ratios. An approximation method can apply in the case of inconsistency.

At the heart of this new book is a paper of Afriat of twenty five years ago in a volume in honour of Sir Richard Stone edited by Angus Deaton, reproduced in Part II. It took being reminded of the paper in a first meeting with Carlo Milana last Christmas (2006) to see what it represents. It is true, a first discovery was agreement about perplexed latter-day phases of the old subject that may deserve notice, perhaps as grim ‘lessons from history’, where one of us, Milana, has a special interest and reports on this subject in Part II. But from then to April this year (2007) three papers were written that take the new method important steps further, and form Part I of this book.

The 1960 paper reproduced in Part II is a step for the proof of a Utility Construction Theorem. This, a half century later, is the “Theorem” of “Two New Proofs of Afriat’s Theorem” by A. Fostel, H. E. Scarf and M. J. Todd. According to the Abstract, it’s “celebrated”!

It happens this paper also serves for another construction theorem where the utility is ‘constant returns’ (already ‘rediscovered’ as if one of us had fallen asleep a few years, even for ever, before being wakened last Christmas) as required by the Utility Cost Factorization Theorem at the start of price index theory proper from a half-century ago, set forth again in Appendix I. There it is also noted how the new method with all its simplicity comes uniquely and unavoidably implied just one step after an application of common sense. In that way it is not just one possible answer to ‘The Index Number Problem’, but the only answer.

Out of this simpler and altogether less well appreciated theorem the new price index method emerged, maybe to be “celebrated” in the next half century, or tomorrow – or, with this publication, even today.

The method should take its place as the unique step that showed the way out of decades of stagnation in the Irving Fisher mind-set

Tracing linkages to the ‘non-parametric’ approach, that since its origin with utility construction theorems has so many influences elsewhere, also makes the basis for the present work.<sup>1</sup>

Though all points in the Paasche-Laspeyres interval are true, all equally, none more true than another, absurd pretensions offer a not merely “true” price index but an extra distinguished “superlative” one, as if it was an important discovery that had escaped everyone, even Irving Fisher, and thicken the quagmire with perambulations about functional form which has no welfare significance at all.

A basis for the method may be the 1981 paper, but only up to a point. That paper was inconsequential for application without the needed computational apparatus, but now we have that. Then there was appeal to the ‘extension property’ of solutions to prove existence theoretically, but now certain solutions are actually computed, the basic price level solutions that come in pairs with sides as it were associated with Laspeyres and Paasche, from which, it is conjectured, all solutions can be derived. A main basis for this book is not altogether in the remote beginnings but in itself.

A precursor is the paper “The Power Algorithm for Generalized Laspeyres and Paasche Indices” at the Athens Meeting of the Econometric Society, September 1979. This deals with a feature in the beginnings of the computational method where a matrix of Laspeyres indices is raised to powers in a bizarre arithmetic where *plus* means *min*, maybe rather much for some but now we have the software for it.

We usually have no dealings with Fisher-type bilateral formulae and the vast literature related there, but for elements of history that include added Notes about chain and multilateral index methods. There is as ever an imbalance since one of us reads everything and the other nothing.

New generation price index workers can have pleasure to escape stagnation in the Irving Fisher trap and catch on to the new approach with its conceptual fidelity and computational elegance.

<sup>1</sup> A bibliography of works with the non-parametric connection lists about two hundred items, and quite likely there are many others.

# Acknowledgements

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# Part I

## Concept and Method